Phase transitions and statistical mechanics for BPS Black Holes in AdS/CFT

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Abstract: Using the general framework developed in hep-th/0607056, we study in detail the phase space of BPS Black Holes in AdS, for the case where all three electric charges are equal. Although these solitons are supersymmetric with zero Hawking temperature, it turns out that these Black Holes have rich phase structure with sharp phase transitions associated to a corresponding critical generalized temperature. We are able to rewrite the gravity variables in terms of dual CFT variables and compare the gravity phase diagram with the free dual CFT phase diagram. In particular, the elusive supergravity constraint characteristic of these Black Holes is particulary simple and in fact appears naturally in the dual CFT in the definition of the BPS Index. Armed with this constraint, we find perfect match between BH and free CFT charges up to expected constant factors.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence.

Contents

1. Introduction

In [1] we developed a framework based on a multi-scaling limit, that defines the "thermodynamics" o better "the statistical mechanics" of supersymmetric solitons in gauge supergravity. One of the basic ideas that grounds that work, is that a supersymmetric partition function can be defined from the general partition function as a combination of limits for the different potentials, but not as the sole naive limit of "temperature → 0", since the BPS equation links all the different charges. Once this framework was settled, as a result of the combination of limits taken new conjugated potentials emerge controlling the resulting BPS charges. These manipulations are easy to implement in a supersymmetric field theory, and amazingly, can also be implemented in supersymmetric configurations of gauge supergravity. Then, as an application, using global and local analysis we showed that BPS Black Holes (BH) present a phase transition as a function of the generalized potentials. For readers interested in the detail explanation of this framework, we reefer to the original article.

In this letter, we continue our studies on statistical mechanics properties of BPS supergravity solitons that were started in a previous set of works [2, 1]. In particular, we expand the above initial studies to describe the phase diagram for BPS BH in AdS. Although we are in a supersymmetric case, we find a sort of instability that translates into a phase transition with a corresponding "generalized critical temperature".[1] Then, we connect our results with the dual CFT picture, to search for a better understanding of the microscopic structure of these BH. We found strong similarities between the supergravity and the free CFT phase diagrams producing a deeper understanding of the supergravity configuration and constraints.

We make notice that the BPS BH studied here are just one of the known families, that is chosen because is the only known solution that has a well behaved off-supersymmetric

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[1]The above physics mimics the well known Hawking-Page transitions at finite temperature of Schwarzschild AdS BH and thermal AdS.

extension. The first solutions were found in [3, 4] and the more general known BPS solutions can be found in [5].

Assuming that the AdS/CFT duality is correct [6, 7], BH in AdS can be understood in the dual CFT theory, as an ensemble of states at strong coupling. In fact, the thermodynamical properties of the dual CFT have been intensively studied for several years, in general the partition function depends on the number of colors N, the canonical ensemble used (like micro canonical, canonical or grand canonical) and the coupling constant. The computation of the partition function in the canonical ensemble at the free regime can be found in [8], extension to include the small couple regime are presented in [9]. Lately the extensions to grand canonical ensemble to include R-charge configurations can be found in [10, 11] and there are works in the literature where approximations of the effective partition function at strong couplings are given [12, 13]. At last, in [14], the supersymmetric partition function with all the relevant chemical potentials turned on, was presented at zero coupling.

BPS BH by analogy, can be related to supersymmetric ensembles at zero temperature but non-zero chemical potentials in the dual CFT. These potential control the expectation value of the pertinent conserved charges carried by the BH, like angular momenta and electric charge. Unfortunately, it is not know how to study the statistical mechanics properties of these ensembles in the dual CFT theory at strong coupling, making very difficult the comparison with the supergravity description. On the other hand, it is possible to study the statistical mechanics of the free CFT theory on a three-sphere at large N, where finite temperature and BPS partition functions have been calculated (see for example [8, 9, 14]). Therefore, *in this note, we work out the strong coupling case using the BPS BH soliton and then, we compare it with statistical mechanics studies at zero coupling in the free CFT theory.*

From these studies, it is reported that there is an amazing similarity between both dual frameworks. To be more concrete, we obtained the BH phase diagram, showing the corresponding phase transition and its interphase region. Also, since is possible to define a generalized potential w_+ conjugated to the energy, we found useful to define a generalized critical potential, as the minimal value of w_+ in the interphase region.

One of the mayor puzzles for these BPS BH is that they come with extra constraints (like extra relation between the conserved charges above the BPS equality) that does not appears in the dual CFT partition function. Somehow, BPS BH corresponds to a particular kind of ensemble i.e. a hypersurface in the general moduli space. Here, by writing the BH generalized potentials in the "natural CFT basis", we discovered that the extra constraint is very simple to write and also has a role in the CFT picture, in the computation of the Index that counts supersymmetric states, defined in [14].

Then, using the free CFT partition function together with the newly found BH constraint, we obtained almost the same phase diagram, and critical potential. The difference lies more in the actual values than in the functional form. In fact, the functional dependence of the resulting BH charges and CFT charges, is the same up to constants in the case where we are well inside the BH/deconfinement phase.

We would like to make notice that other studies at finite temperature like [8] indicate

that the statistical mechanics on three-sphere has a smooth dependence on the coupling. We believed that also in this case, on the top of the BPS character of the sector under study this smooth dependence reappears and is the underlying reason for the reported similarities.

This work is organized as follows: In section 2 we borrow from [1], the necessary information to define the relevant statistical mechanics studies of BPS BH. We then elaborate further to define the generalized critical potential and describe the phase diagram for this BPS sector. In section 3, we study the CFT dual ensemble by means of the free BPS partition function together with the constraint found in the previous section. Then, the corresponding critical potential and phase space are described. Also the form of the charges in both dual theories is compared well inside the BH/deconfinement phase. In section 4, we comment the results, making some conclusions and final remarks on future research and open problems.

2. The strong coupling case (supergravity)

We start this section with a short overview of the BH solutions of minimal gauge supergravity in five dimensions of [15]. In general, the solutions is characterized by its energy E, two independent angular momenta (J_1, J_2) and a single electric charge Q. In the BPS regime, the solutions preserve only a fraction of 1/16 out of the total 32 supercharges of the uplifted ten dimensional type IIB supergravity and depending on the different range of values of its parameter space, the solutions describe BPS BH or topological solutions with no horizon (here we will concentrate in the BH case only).

The form of the solution can be found in [15] while in [1], it is defined and explicitly calculated the multi-scaling limit necessary to study the statistical mechanics of the solution, in particular it is showed how to define the BPS charges $(Q_{\mathrm{bps}}, J_{\mathrm{bps}}^1, J_{\mathrm{bps}}^2)$, its generalized potentials (ϕ, w_1, w_2) and the entropy S_{bps}. At the BPS bound the different charges satisfy that

$$E_{\mathrm{bps}} = \sqrt{3}\, Q_{\mathrm{bps}} + J_{\mathrm{bps}}^1, + J_{\mathrm{bps}}^2 .$$

Here, to avoid a rather long discussion, we show the final expressions referring to the original articles for details. First we present the charges and entropy

$$E_{\mathrm{bps}} = \frac{\pi(a+b)\left[(1-a)(1-b) + (1+a)(1+b)(2-a-b)\right]}{4(1-a)^2(1-b)^2} ,$$

$$J_{\mathrm{bps}}^1 = \frac{\pi(a+b)(2a+b+ab)}{4(1-a)^2(1-b)} ,$$

$$J_{\mathrm{bps}}^2 = \frac{\pi(a+b)(a+2b+ab)}{4(1-a)(1-b)^2} ,$$

$$Q_{\mathrm{bps}} = \frac{\sqrt{3}\pi(a+b)}{4(1-a)(1-b)} ,$$

$$S_{\mathrm{bps}} = \frac{\pi^2(a+b)r_0}{2(1-a)(1-b)} .$$

Second, the conjugated generalized potentials

$$w_1 = \frac{\pi(1-a)(a+2ab+b^2+2b)}{r_{\text{bps}}(3r_{\text{bps}}^2+1+a^2+b^2)},$$

$$w_2 = \frac{\pi(1-b)(b+2ab+a^2+2a)}{r_{\text{bps}}(3r_{\text{bps}}^2+1+a^2+b^2)},$$

$$\phi = \frac{\pi\sqrt{3}(a+b)(1-ab)}{r_{\text{bps}}(3r_{\text{bps}}^2+1+a^2+b^2)}.$$

where $r_{\text{bps}}^2 = a + b + ab$, and $(a, b) \leq 1$. Notice that all the above quantities come as a function of only two parameters (a, b).

These generalized potentials where defined in [1], as the next-to-leading term in the multi-scaling limit that defines the BPS solution, of the well known potentials of BH thermodynamics. In fact, the explicit definition is given by

$$\beta \to \infty, \quad \Omega \to \Omega_{\text{bps}} - \frac{w}{\beta} + O(\beta^{-2}), \quad \Phi \to \Phi_{\text{bps}} - \frac{\phi}{\beta} + O(\beta^{-2}).$$

where (β, Ω, Φ) are respectively the inverse Hawking temperature, angular velocity of the horizon and electric potential of the general off-BPS BH solution under study.

With these definitions we are ready to define the Quantum Statistical Relation (QSR)

$$I_{\text{bps}} = \phi\, Q_{\text{bps}} + w_1 J_{\text{bps}}^1 + w_2 J_{\text{bps}}^2 - S_{\text{bps}}$$

where I_{bps}, is the value of the supersymmetric Euclidean action of the corresponding BH. Solving for the explicit form of these different quantities in the BH solution, we obtain

$$I_{\text{bps}} = \frac{\pi^2(a+b)^2[-1+2b+b^2+b^2+a(2+5b+b^2)+a^2(1+b)]}{4(1-a)(1-b)\sqrt{a+b+ab}(1+a^2+b^2+3(a+b+ab))}.$$

The range of the parameters (a, b) is obtained by imposing that the event horizon radius r_{bps} and the energy are real positive expression. In figure 1 and figure 2 we show two different plots of I_{bps}, where in the first case, I_{bps} is a function of (a, b) while in the second I_{bps} is at fixed $b = .1$ and running a. As it was found in [1], it is easy to see that I_{bps} is positive for small (a, b) and negative for larger values. Therefore we deduce that *Indeed there is a phase transition*, where the BH solution is not any more the preferred vacuum, but a meta-stable vacuum. The stable vacuum most probably is a gas of superparticles in AdS, studied in detail in [14].

At this point of the analysis, we found more convenient to make a change of variables that facilitates the future confrontation with CFT results of the next section. Then, we use the following

$$J^{\pm} = J_{\text{bps}}^1 \pm J_{\text{bps}}^2, \qquad E = J_{\text{bps}}^1 + J_{\text{bps}}^2 + \sqrt{3}Q_{\text{bps}}, \qquad Q = Q_{\text{bps}}$$

$$w_{\pm} = \frac{(w_1 \pm w_2)}{2}, \qquad \lambda = (\phi - \sqrt{3}w_+), \qquad S = S_{\text{bps}}.$$

In this new set of variables the QSR is given by

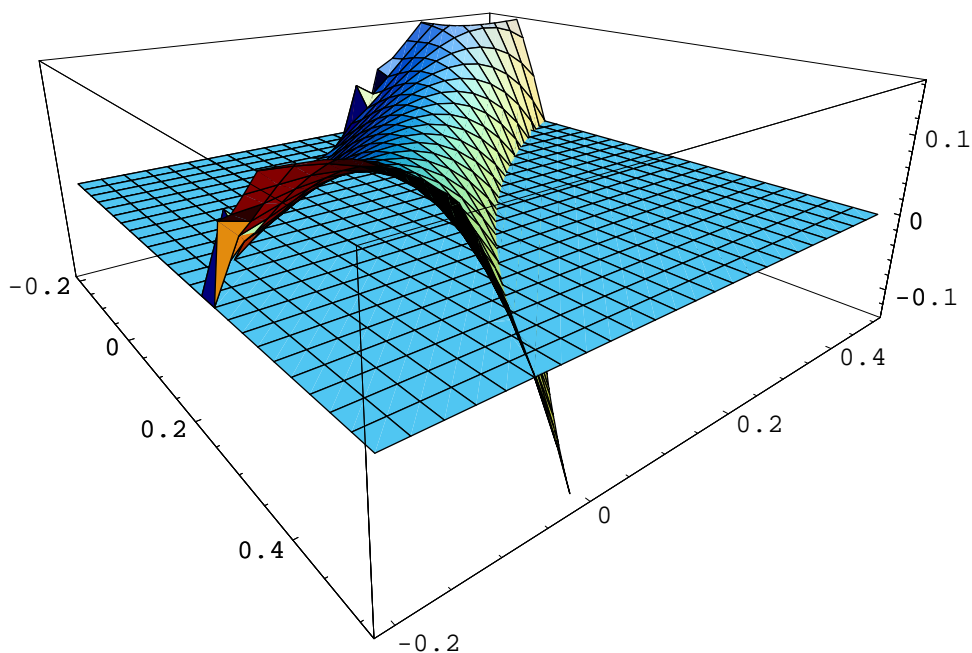$$I_{\text{bps}} = w_+ E + w_- J^- + \lambda Q - S.$$

Figure 1: Plot of the Euclidean action of the BPS BH as a function of the parameters (a, b). The flat plane corresponds zero level surface.
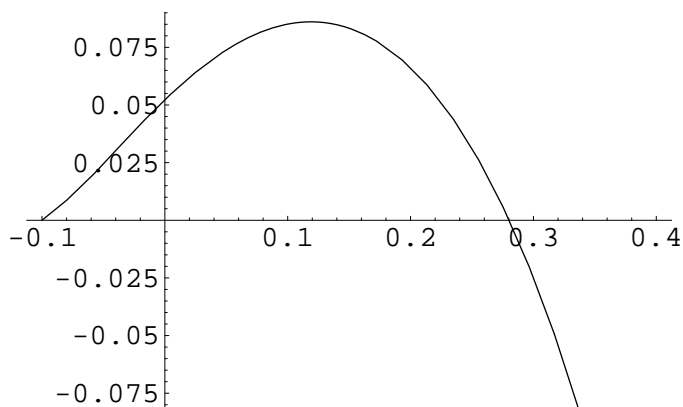

Figure 2: Plot of the Euclidean action at fixed $b = 1/10$.

Some of the advantages of these new variables is that we have obtained a potential conjugated to the energy (a sort of generalized inverse of the temperature) w_+ and that the Euclidean action in not a functional of J^+. Also, we get the new left an right angular momenta (J^+, J^-) and the generalized potentials (w_-, λ) conjugated to (J^-, Q) respectively.

Notice that, in principle we should have three independent parameters (four charges (E, J^+, J^-, Q) modulo the BPS bound). Instead our expressions come as functions of only two parameters, showing that these BH are constraint systems. We have found that the
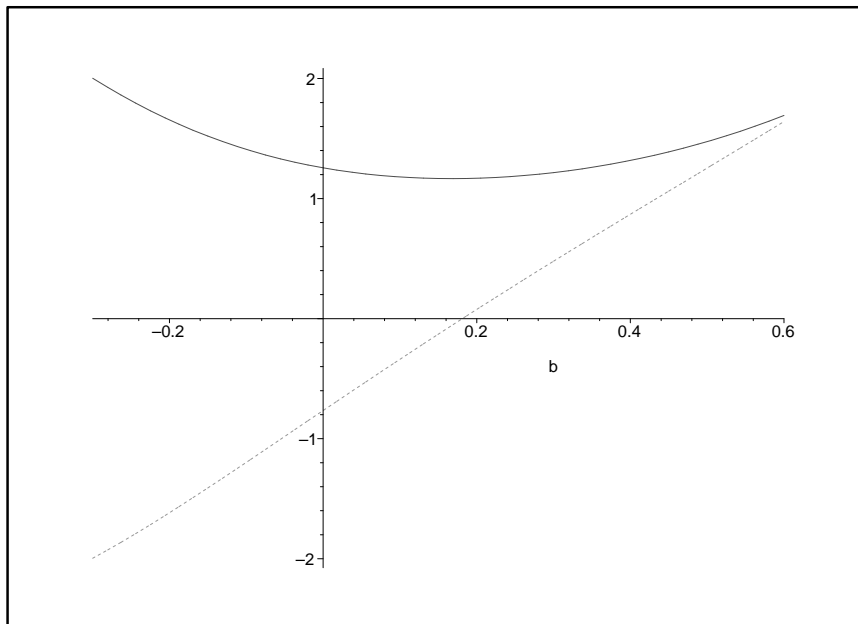
Figure 3: Plot of w_+ and w_- at the phase transition, as a function of b. w_+ is plotted with a continuous line, while w_- is in dotted line.

corresponding constraint is amazingly simple in this set variables, namely that

$$\lambda(a,b) = -\frac{w_+(a,b)}{\sqrt{3}} \, .$$

In other words, we are looking into BH solutions where two out of the three generalized potentials are proportional! In essence, we have only two degrees of freedom, that we will choose to be (w_+, w_-) for the rest of the analysis with no lose of generality.

Next, to better characterized the above phase transition, we first identify its locus, where $I_{\text{bps}} = 0$. After some algebra the above requirement reduces to a quadratic equation with the two associated roots

$$a(b)_{\mp} = \frac{(-b^2 + 5b + 2) \mp \sqrt{b^4 + 2b^3 + 13b^2 + 16b + 8}}{2(1+b)} \, .$$

We found that $a(b)_-$ is not a physical solution since in this range of parameters $(a(b)_-, b)$, the event horizon radius r_{bps} is pure imaginary. The other solution $a(b)_+$ is physical and in fact corresponds to the phase transition observed before. Then, in figure 3 we show a plot of the two generalized potentials at the phase transition locus as a function of the parameter b. It is not difficult to evaluate numerically all three generalized potentials (w_+, w_-, λ), at the maximum of the critical generalized temperature (minimum of w_+), obtaining

$$w_+ = 1.1668 \quad , \quad \lambda = -0.673 \quad , \quad w_- = 0 \, . \tag{2.1}$$

These values correspond to the point where $a = b \approx 0.1813$. That w_+ is at its minimum when w_- is zero, is expected since we are looking for the maximal generalized critical
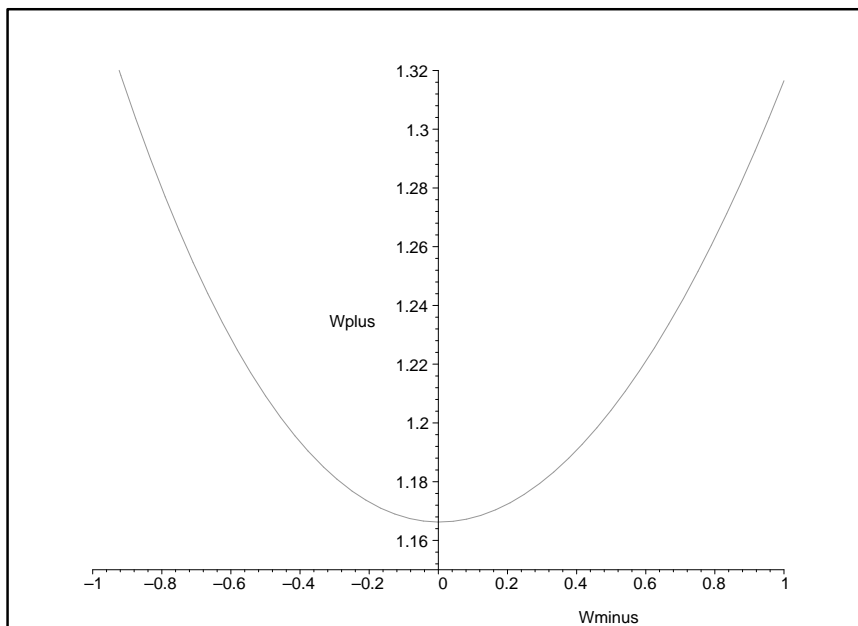
Figure 4: Plot of w_+ as a function of w_-, the curve shows the phase transition.

temperature. In fact, at this point there is no J^- charge and we need more energy i.e. less w_+ to obtain a phase transition.

Unfortunately, we could not solve for b, to rewrite the phase diagram in terms of (w_+, w_-) alone in a analytic form. Nevertheless, after some inspection it is clear that w_- is almost a linear function of b in the vicinity of the minima for w_+. Based on this observation we used the approximation that $w_- \approx 4.3b - .766$ to solve for b and then draw the final version of the phase diagram figure 4. In this plot, it can clearly be seen the phase transition diagram as a function of the two generalized potentials (w_+, w_-) conjugated to the two charges (E, J^-). The region in the exterior of the curve, corresponds the BH phase, while the region in the interior of the curve corresponds to the AdS phase.

3. The zero coupling case (free CFT)

In this section, we study the same supersymmetric sector of the above section, but this time in the dual picture using the conformal field theory language. The ideal situation would be to calculate the CFT partition function in the strong coupling regime and then compare its structure against the supergravity results. Unfortunately, such calculations are not within our actual capabilities and therefore we have decided to work instead with the simpler case where the partition function is known, corresponding to the free theory i.e. at zero coupling.

Before starting the actual calculations, let us recall that similar studies at finite temperature resulted in a celebrated connection between the Hawing-Page phase transition (Schwarzschild AdS BH and thermal AdS) and the deconfinement/confinement transition of large N, strongly coupled $\mathcal{N}=4$ Super Yang Mills theory on a three sphere [16, 17]. In the present case, we will work in the grand canonical ensemble with all possible potentials

turned on (since we are interested in configurations with generically non-zero expectation value for angular momenta and electric charge), focussing on the partition function over BPS states that is associated to zero temperature statistical mechanics.

As it is well known, $\mathcal{N}=4$ Super Yang Mills at zero coupling on a three-sphere is simple enough for explicit calculations. We are dealing with free dynamics in the presence of a global constraint of neutral color in all states (related to Gauss law on compact manifolds). In [14], the form of the partition function relevant to our studies was computed (see also [8, 9] for general calculations at non-zero temperature). If we start with the definition of the supersymmetric partition function, constrained to the case where all three R-charges are set equals,[2] we first write

$$Z = \sum_{\text{bps}} e^{-\beta e + 3\mu q + 2\xi j_-} \,, \tag{3.1}$$

that defines the conjugated potentials (β, μ, ξ) to the charges (e, q, j_-) respectively energy, R-charge and angular momentum. Then it can be shown that

$$Z = \int DU \exp \left\{ \sum_n \left[f_B(x^n, y^n, v^n) + (-1)^{n+1} f_F(x^n, y^n, v^n) \right] \frac{Tr U^n U^{-n}}{n} \right\},$$

where U is a unitary matrix, $x = e^{-\beta}, y = e^{\mu}, v = e^{\xi}$,

$$f_B = \frac{3yx + x^2}{(1 - xv)(1 - x/v)}, \quad f_F = \frac{x^{3/2} \left[(v + 1/v) y^{3/2} + 3y^{1/2} \right] - x^{5/2} y^{3/2}}{(1 - xv)(1 - x/v)},$$

In [14], it was found that the above partition function undergoes a phase transition at finite values of the generalized potentials, where one phase is independent of N, while the other phase goes like N^2.

The phase transition can be studied searching for the singular behavior of the partition function Z. The locus of the phase transition (i.e. the generalized critical surface), is found by the strongest singularity of Z i.e.

$$f_B(x^n, y^n, v^n) + (-1)^{n+1} f_F(x^n, y^n, v^n) = 1 \,, \tag{3.2}$$

that corresponds to the case $n = 1$.

At first sight, it is easy to see that our partition function depends on too many variables to successfully reproduce the supergravity results. Recall that the supergravity BH depends on only two parameters and hence would corresponds to a particular class of ensemble within the general ensemble of (3.1). Therefore, we have to find somehow a constraint to reduce the number of independent generalized potentials from three to two.

At present, the nature of this constraint is by no means clear in the CFT picture (see [18] for some clues). Nevertheless in the previous section we learned how the supergravity generalized potentials where related by the equation $\lambda = -w_+/\sqrt{3}$. Therefore, we

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[2]We are considering only the case of all three R-charges equals i.e. $q = q_1 = q_2 = q_3$. Also we have changed by little the original notation of [14], in order to accommodate better our previous section.
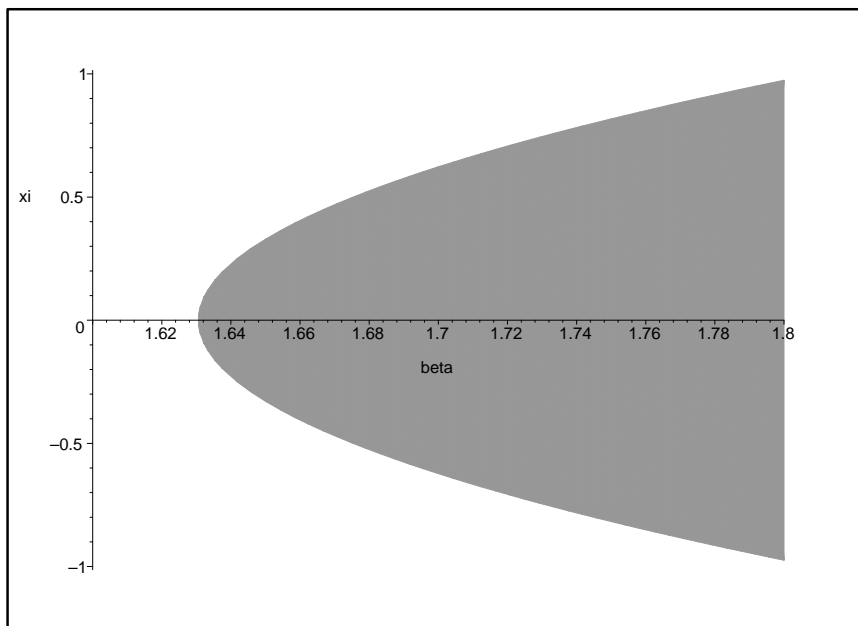
Figure 5: Plot of the phase space with the line of interface. In the plot, ξ runs along the vertical axes, while β runs in the horizontal axes. The shaded region corresponds to the region where Z is independent of N.

found natural to impose this relation upon the CFT potential to check its implication and results. To implement this constraint in the CFT picture, we just have to notice that the normalization of the potential μ and its charge q is of, by a factor of $\sqrt{3}$ when compared to the supergravity definitions. Once this is taken into account the constraint in CFT variables reads

$$\mu = -\beta/3 \tag{3.3}$$

Note that this is the same relation found in [14], for the definition of the Index. Off course, in the partition function Z, there is no cancellation between configurations due to the extra factor $(-1)^F$ characteristic of the Index.

Therefore with this constraint the locus of interphase is defined by

$$\frac{\left[3x^{\frac{4}{3}} + (1 + v + 1/v)x^2 + 3x^{\frac{10}{6}} - x^3\right]}{(1 - xv)(1 - x/v)} = 1 \,.$$

At this point, it is not difficult to find the value of the minimal generalized critical potential β and the corresponding values of the other two potentials,

$$\beta \approx 1.6301 \quad , \quad \mu \approx -0.5435 \quad , \quad \xi = 0 \,. \tag{3.4}$$

Using the constraint partition function, we can in principle obtain the phase diagram as a function of (β, ξ). For technical reasons we found more easy to solve for ξ as a function of β, and then obtain numerically the desiderate free large N CFT phase diagram. The resulting phase diagram is showed in figure 5.

Note that the shape of the diagram matches very well the supergravity case, showing that the free theory calculation is a valuable regime where to look for BH physics. Regarding the generalized critical potential, note that the free value is bigger than the strongly coupled value and this can be explain since in the free regime, there are "more" BPS states than in the strongly couple regime and therefore we need less "temperature" to create a BH out of a thermal ensemble.

It is interesting to cross check our results with an observation made in [14], where the expectation values of (e, j_-, q) were used to guess a constraint in the particular regime of the moduli space well inside the BH phase. There, it was found $\mu \approx -0.504$. Here, we can use this value of μ into eq. (3.2) to calculate the corresponding generalized Hagedorn potential β (assuming therefore a constant μ as a rough approximation). The result is $\beta \approx 1.65$ at $\xi = 0$ and hence gives a value that is still larger than the strong coupling result of eq. (2.1) but also larger than our zero coupling result of eq. (3.4). Therefore (3.4) is closer to the strong coupling value that is consistent with the idea that our constrain is more accurate.

Another important check is to compare the form of the resulting charges in both dual descriptions. In general points of the phase space, the CFT expressions for the charges are too complicated but in [14], it was found that things get more manageable if we are well inside the BH phase. There, it was used that $(\beta, \xi) \ll 1$, while μ was left free.

In this regime, the explicit expressions for the energy, electric charge, angular momentum and entropy (respectively (e, q, j_-, s)) are

$$
e = \frac{2\beta f(\mu) N^2}{(\beta^2 - \xi^2)^2}, \qquad 2j_- = \frac{2\xi f(\mu) N^2}{(\beta^2 - \xi^2)^2},
$$
$$
q = \frac{g(\mu) N^2}{(\beta^2 - \xi^2)}, \qquad s = \frac{(3f(\mu) - \mu g(\mu)) N^2}{(\beta^2 - \xi^2)},
$$
(3.5)

where $f(\mu) = (\zeta(3) + 3Pl(3, y) - 3Pl(3, -y^{1/2}) - Pl(3, -y^{3/2}))$, $g(\mu) = \partial_\mu f / 3$, $Pl(s, z)$ is the PoliLog function and $\zeta(n)$ is the Riemann's Zeta function. Notice now, that if we use our constraint (3.3), $\mu \ll 1$, and therefore the above expression become even more simple reducing to

$$
e = \frac{14\zeta(3)\beta N^2}{(\beta^2 - \xi^2)^2}, \qquad 2j_- = \frac{14\zeta(3)\xi N^2}{(\beta^2 - \xi^2)^2},
$$
$$
q = \frac{\pi^2 N^2}{4(\beta^2 - \xi^2)}, \qquad s = \frac{21\zeta(3) N^2}{(\beta^2 - \xi^2)},
$$
(3.6)

where we have used that $f(0) = 7\zeta(3)$ and $g(0) = \pi^2/2$.

This results are to be compared with the BH charges in a corresponding region of the parameter space, namely where $a = 1 - (\tilde\beta + \tilde\xi)$ and $b = 1 - (\tilde\beta - \tilde\xi)$. The final expressions for the charges is

$$
E = \frac{8\tilde\beta N^2}{(\tilde\beta^2 - \tilde\xi^2)^2}, \qquad 2J_- = \frac{8\tilde\xi N^2}{(\tilde\beta^2 - \tilde\xi^2)^2},
$$
$$
Q = \frac{\sqrt{3} N^2}{(\tilde\beta^2 - \tilde\xi^2)}, \qquad S = \frac{2\pi\sqrt{3} N^2}{(\tilde\beta^2 - \tilde\xi^2)},
$$
(3.7)

where the factor $\sqrt{3}$ of Q is due to the different normalization between Q and q. Notice that both relations are functionally identical and the only difference lies in the value of the different constants. The fact that the CFT and the dual BH expression are so similar for the case $\mu \ll 1$ was noted in [14], the difference is that here we obtain this limit as the necessary condition once the constraint 3.3 is used.

Therefore, in spite all the difference that characterize the supergravity and the free CFT frameworks, we have found in this work strong similarities between the two dual descriptions even so they are calculated at such different regimes on the coupling constant. We suspect that the match of the structure is not only linked to supersymmetry but may also indicate that the statistical mechanics of the CFT on the three-sphere, has a smooth dependence on the coupling constant.

4. Discussion

In this work, we have studied the statistical mechanics properties of BPS BH of minimal gauge supergravity in five dimensions. In order to carry on these studies, we used the new framework defined in [1]. Then, based on the AdS/CFT duality, we contrast our results with CFT calculations using more standard statistical mechanics methods developed in [14].

As main conclusion we point out that these BH present a rich phase structure with phase transitions and generalized critical potential. The results tell us about the phase structure of the dual CFT theory at strong coupling. Also, we found that the free theory has strong similarities with the strong coupling case, since in this regime, the CFT presents the same king of phase diagrams and only constant factors seems to be different.

This work does not exhaust all the physical structure of the BPS phase space. In particular we have only compared the supergravity results with the free CFT picture, but it is well known the things may change in the interacting theory even a very weak coupling. Therefore we think it will be vary interesting to study the weakly and strongly coupled supersymmetric CFT partition function (see $[9-11, 13, 12, 19]$ for some extension in the non-supersymmetric case along this directions).

At a more technical level, we found that the BH ensemble is characterized by a simple constraint, acting on the generalized potentials, that also appears in the CFT theory. Unexpectedly, in the CFT theory this constraint shows up only in the construction of an Index and not in the partition function [14]. Apparently, these BH obey strict relations of group theory representation that we believe should be utilized to guess properties of the corresponding microstates. Nevertheless, at present we have not a clear picture of the above.

It will be very interesting to study more general BPS Black holes or/and possibly Black Rings, but unfortunately all the known solutions are either singular in the BPS regime or not known out of the BPS regime. As a consequence of the above, both cases we can not be studied with the framework of [1], since we need well behaved solutions to consider the multi-scaling limit. We are currently working in a generalization to cover these singular cases too [20].

Also, we point out that in [14] it was found another type of phase transition more alike to a Bose-Einstein condensation in more supersymmetric sectors of the CFT. It will be very interesting to see what is the dual description of such phenomena in supergravity.

At last, in retrospective, this work may be seen as a confirmation that the general framework developed in [1] is correct and therefore should be useful to better understand the microscopic structure of BH and quantum gravity.

Acknowledgments

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